WORKSHEET FOR INSTRUCTORS
RATIONAL EQUATIONS

1. $x + \frac{6}{x} = -7$
2. $x + 5 = \frac{14}{x}$
3. $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$
4. $1 = \frac{n-2}{n-1} + \frac{3}{n^2+3n-4}$
5. $1 = \frac{2}{r^2} - \frac{1}{r}$
6. $\frac{5}{x+1} = \frac{6}{x^2-2x-3} + \frac{1}{x-3}$
7. $\frac{1}{x} = \frac{6}{5x} + 1$
8. $1 + \frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$
9. $\frac{x^2-3x-4}{x^3-x^2} - \frac{1}{x^2} = \frac{x-2}{x^2}$
10. $\frac{5}{p+6} - \frac{1}{p^2+6p} = \frac{2}{p^2+6p}$

RATIONAL EQUATIONS
(SOLUTION FOR INSTRUCTORS)

1) $x+\frac{6}{x}= -7$

$= \frac{x}{1}+\frac{6}{x}=\frac{-7}{1}$ **(1)**

$\text{LCD }(1,x,1) = x$

Multiply (1) by x

$\left(\frac{x}{1}+\frac{6}{x}=\frac{-7}{1}\right)x$ ⇨ $x^2+6=-7x$

$x^2+7x+6=0$ ⇨ $(x+1)(x+6)=0$ ⇨ $x=-1$ or $x=-6$

Check for x= -1

Substitute $x-1$ in $x+\frac{6}{x}= -7$

$-1+\frac{6}{-1}= -7$ ⇨ -7 = -7

So, $x=-1$ is a Solution.

Check for x= -6

Substitute $x=-6$ in $x+\frac{6}{x}= -7$

$-6+\frac{6}{-6}= -7$ ⇨ -6-1= -7 ⇨ -7=-7

So, $x=-6$ is a Solution.

2) $x+ 5=\frac{14}{x}$

⇨ $\frac{x}{1}+\frac{5}{1}=\frac{14}{x}$ **(1)**

$\text{LCD}(1, 1, x) = x$

Multiply (1) by the x

$\left(\frac{x}{1}+\frac{5}{1}=\frac{14}{x}\right)x$ ⇨ $x^2+5x=14$

⇨ $x^2+5x-14=0$

= $(x-2)(x+7)=0$ ⇨ $x=2$ or $x=-7$

Check for x= 2

Substitute $\mathrm{x} = 2$ in $\mathrm{x} + 5 = \frac{14}{\mathrm{x}}$

2+5=$\frac{14}{2}$ ⇨ 7=7

So, $\mathrm{x} = 2$ is a solution.

Check for x = - 7

Substitute $\mathrm{x} = -7$ in $\mathrm{x} + 5 = \frac{14}{\mathrm{x}}$

-7+5=$\frac{14}{-7}$ ⇨-2 =-2

So, $\mathrm{x} = -7$ is a Solution.

3) $\frac{3}{x+2}+\frac{6}{x^2+2x}=\frac{3-x}{x+2}$

Factoring we get $\frac{3}{x+2}+\frac{6}{x(x+2)}=\frac{3-x}{x+2}$ $\quad$ **(1)**

LCD$(x + 2, x(x + 2), x) = x(x+2)$

Multiply (1) by the $x(x + 2)$

$$\left(\frac{3}{x+2}+\frac{6}{x(x+2)} = \frac{3-x}{x}\right)x(x+2)$$

⇨$3x+6 = (3-x)(x + 2)$

⇨ $3x + 6 = 3x + 6 - x^2 - 2x$

⇨$x(x + 2) = 0$

⇨ $x = 0$ or $x = -2$

Check for x= 0

Substitute $x = 0$ in the equation $\frac{3}{x+2}+\frac{6}{x^2+2x} = \frac{3-x}{x}$

This will give a zero denominator because of the presence of $\frac{3-x}{x}$

Check for x= -2

Substitute $x = -2$ in $\frac{3}{x+2}+\frac{6}{x^2+2x}=\frac{3-x}{x}$

Here too we get a zero denominator because of the presence of $\frac{3}{x+2}$ and $\frac{6}{x^2+2x}$.

So, both x=0 and $x = -2$ are extraneous Solution.

So, the equation $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$ has no real Solutions.

4) $1 = \frac{n-2}{n-1} + \frac{3}{n^2+3n-4}$

After factoring we get,

$$1 = \frac{n-2}{n-1} + \frac{3}{(n+4)(n-1)} \qquad \mathbf{(1)}$$

LCD $[(n-1), (n+4)(n-1)] = (n-1)(n+4)$

Multiply (1) by $(n-1)(n+4)$

$$\left(1 = \frac{n-2}{n-1} + \frac{3}{(n+4)(n-1)}\right)(n-1)(n+4)$$

$(n-1)(n+4) = (n-2)(n+4) + 3$

$n^2 + 3n - 4 = n^2 + 2n - 8 + 3$

⇨ $3n - 4 = 2n - 5$ ⇨ $n = -1$

Check for $n = -1$

Substitute n = -1 in the equation $1 = \frac{n-2}{n-1} + \frac{3}{n^2+3n-4}$

$1 = \frac{-1-2}{-1-1} + \frac{3}{(-1)^2+3(-1)-4}$

⇨ $1 = \frac{3}{2} + \frac{3}{-1-3-4}$

⇨ $1 = \frac{3}{2} + \frac{3}{-6}$

⇨ $1 = \frac{3}{2} - \frac{1}{2}$

⇨ 1=1

So, $n = -1$ is a Solution.

5). $1 = \frac{2}{r^2} - \frac{1}{r}$

$$\frac{1}{1} = \frac{2}{r^2} - \frac{1}{r} \qquad \mathbf{(1)}$$

$LCD(1, r^2, r) = r^2$

Multiply(1)by r^2

$$\left(1 = \frac{2}{r^2} - \frac{1}{r}\right) r^2$$

r^2= 2 - r ⇨ r^2+r-2=0

$(r+2)(r-1) = 0$ ⇨ r=-2 or r=1

<u>Check for r = -2</u>

Substitute r = -2 in 1 = $\frac{2}{r^2} - \frac{1}{r}$

1 = $\frac{2}{(-2)^2} - \frac{1}{-2}$

⇨1 = $\frac{1}{2} + \frac{1}{2}$

⇨ 1=1

So, r=-2 is a Solution.

<u>Check for r = 1</u>

Substitute r=1 in 1=$\frac{2}{r^2} - \frac{1}{r}$

1=$\frac{2}{(1)^2} - \frac{1}{1}$ ⇨

1 = 2-1

⇨ 1=1

So, r=1 is a Solution.

6) $\frac{5}{x+1} = \frac{6}{x^2-2x-3} + \frac{1}{x-3}$

After factoring

$$\frac{5}{x+1} = \frac{6}{(x-3)(x+1)} + \frac{1}{x-3} \qquad (\mathbf{1})$$

LCD$[(x+1), (x-3)(x+1), (x-3)]$=$(x+1)(x-3)$

Multiplying (1) by $(x+1)(x-3)$

$$\left(\frac{5}{x+1} = \frac{6}{(x-3)(x+1)} + \frac{1}{x-3}\right)(x+1)(x-3)$$

$5(x-3) = 6 + x + 1$

⇨ $5x - 15 = x + 7$

$4x = 22$

⇨ $x = \frac{22}{4}$

$\Rightarrow x = \frac{11}{2}$

Check for x = $\frac{11}{2}$

Substitute $x = \frac{11}{2}$ in $\frac{5}{x+1} = \frac{6}{x^2-2x-3} + \frac{1}{x-3}$

$\frac{5}{\frac{11}{2}+1} = \frac{6}{(\frac{11}{2})^2-2(\frac{11}{2})-3} + \frac{1}{\frac{11}{2}-3}$

$\frac{10}{13} = \frac{6}{\frac{65}{4}} + \frac{2}{5}$

$\Rightarrow \frac{10}{13} = \frac{24}{65} + \frac{2}{5}$

$\Rightarrow \frac{10}{13} = \frac{50}{65}$

$\Rightarrow \frac{10}{13} = \frac{10}{13}$

So, $x = \frac{11}{2}$ is a Solution

7) $\frac{1}{x} = \frac{6}{5x} + 1$

$\Rightarrow \frac{1}{x} = \frac{6}{5x} + \frac{1}{1}$ $\quad (\mathbf{1})$

LCD $(x, 5x, 1) = 5x$

Multiply (1) by 5x

$\left(\frac{1}{x} = \frac{6}{5x} + \frac{1}{1}\right)5x$

$\Rightarrow 5 = 6 + 5x$

$\Rightarrow x = \frac{-1}{5}$

Check for x= $\frac{-1}{5}$

Substitute $x = \frac{-1}{5}$ in the equation $\frac{1}{x} = \frac{6}{5x} + 1$

$\frac{1}{\frac{-1}{5}} = \frac{6}{5\left(\frac{-1}{5}\right)} + 1$

$-5 = -6 + 1 \Rightarrow -5 = -5$

So, $x = \frac{-1}{5}$ is a solution.

8) $1+\frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$

$\frac{1}{1}+\frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$ $\quad(\mathbf{1})$

$LCD(1,3x,3x) = 3x$

Multiply (1) by 3x.

$\left(\frac{1}{1}+\frac{x^2-5x-24}{3x} = \frac{x-6}{3x}\right)3x$

$3x+\ x^2-5x-24 = x-6$

⇨$x^2-2x-24 = x-6$

⇨ $x^2-3x-18 = 0$

⇨ $(x+3)(x-6) = 0$

$x = -3$ or $x = 6$

<u>Check for $x = -3$</u>

Substitute $x = -3$ in the equation $1+\frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$

$1-\frac{(-3)^2-5(-3)-24}{3(-3)} = \frac{-3-6}{3(-3)}$

⇨ 1+0= $\frac{-9}{-9}$

⇨1=1

So, $x = -3$ is a Solution.

<u>Check for $x = -6$</u>

Substitute $x = -6$ in $1+\frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$

$1+\frac{(6)^2-5(6)-24}{3(6)} = \frac{6-6}{3(6)}$

⇨ 1+$\frac{(-18)}{18}$= 0

⇨0=0

So, $x = 6$ is a Solution.

9) $\frac{x^2-3x-4}{x^3-x^2} - \frac{1}{x^2} = \frac{x-2}{x^2}$

After factoring we get,

$\frac{x^2-3x-4}{x^2(x-1)} - \frac{1}{x^2} = \frac{x-2}{x^2}$

LCD$[x^2(x-1), x^2, x^2] = x^2(x-1)$

Multiplying $(1)$by $x^2(x-1)$

$$\left[\frac{x^2-3x-4}{x^2(x-1)} - \frac{1}{x^2} = \frac{x-2}{x^2}\right]x^2(x-1)$$

x^2-3$x - 4 - (x-1) = (x-2)(x-1)$

x^2-3$x - 4 - x + 1 = x^2$- 3$x + 2$

x^2-4$x - 3 = x^2$-3$x + 2$

⇨ -4$x - 3 = -3x + 2$

⇨ $x = -5$

Check for x=-5

Substitute x = -5 in the equation $\frac{x^2-3x-4}{x^3-x^2} - \frac{1}{x^2} = \frac{x-2}{x^2}$

$\frac{(-5)^2-3(-5)-4}{(-5)^3-(-5)^2} - \frac{1}{(-5)^2} = \frac{-5-2}{(-5)^2}$

$\frac{25+15-4}{-125-25} - \frac{1}{25} = \frac{-7}{25}$

⇨$\frac{-36}{150} - \frac{1}{25} = \frac{-7}{25}$

⇨$\frac{-42}{150} = \frac{-7}{25}$ ⇨ $\frac{-7}{25} = \frac{-7}{25}$

So, x = -5 is a solution

10. $\frac{5}{p+6} - \frac{1}{p^2+6p} = \frac{2}{p^2+6p}$

After factoring we get

$\frac{5}{p+6} - \frac{1}{p(p+6)} = \frac{2}{p(p+6)}$ $\quad\quad\quad (\mathbf{1})$

LCD $[(p + 6, p(p + 6), p(p + 6)] = p(p + 6)$

Multiply (1) by p(p+6)

$\left(\frac{5}{p + 6} - \frac{1}{p(p + 6)} = \frac{2}{p(p + 6)}\right) p(p + 6)$

5p-1 = 2 ⇨ p=$\frac{3}{5}$

Check for p = $\frac{3}{5}$

Substitute p=$\frac{3}{5}$ in equation $\frac{5}{p+6} - \frac{1}{p^2+6p} = \frac{2}{p^2+6p}$

$\frac{5}{\frac{3}{5}+6} - \frac{1}{(\frac{3}{5})^2+6(\frac{3}{5})} = \frac{2}{\frac{3}{5}^2+6(\frac{3}{5})}$

$\frac{5}{\frac{33}{5}} - \frac{1}{\frac{99}{25}} = \frac{2}{\frac{99}{25}}$

$\frac{25}{33} - \frac{25}{99} = \frac{50}{99}$

⇨$\frac{50}{99} = \frac{50}{99}$

So, p = $\frac{3}{5}$ is a Solution.